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Choosing the best momentum for K^+ mass measurement

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Abstract

I make an argument for why it is better to take data at lower momentum in order to make a more accurate measurement of the kaon mass with the RICH. Based on this estimate 40 GeV/c is probably the optimal momentum, and with at 10^7 events MIPP should be capable of making a good measurement of kaon mass.

1 Calculation

Suppose that we only use pion and kaon ring radii in order to do the measurement. Given

$$\theta_\pi^2 - \theta_K^2 = \frac{m_K^2 - m_\pi^2}{p^2},$$

where θ_π and θ_K are Cherenkov opening angles for pion and kaon respectively (see MIPP note 53), we get

$$m_K^2 = m_\pi^2 + p^2(\theta_\pi^2 - \theta_K^2). \tag{1}$$

Now all we have to do is propagate errors. I assume that the error on pion mass is much smaller than the error on momentum measurement or the measurement of the ring radii. Partial derivatives give us:

$$\begin{aligned}
\frac{1}{2}\frac{\partial}{\partial p} m_K^2 &= p(\theta_\pi^2 - \theta_K^2) = 2\frac{m_K^2 - m_\pi^2}{p} \\
\frac{1}{2}\frac{\partial}{\partial \theta_\pi} m_K^2 &= p^2\theta_\pi \\
\frac{1}{2}\frac{\partial}{\partial \theta_K} m_K^2 &= p^2\theta_K
\end{aligned}$$

Therefore, if we define $\Delta_{K\pi} \equiv m_K^2 - m_\pi^2$,

$$\frac{1}{4}\sigma^2_{m_K^2} = \Delta^2_{K\pi}\left(\frac{\sigma_p}{p}\right)^2 + p^4\left(\theta_\pi^2\sigma^2_{\theta_\pi} + \theta_K^2\sigma^2_{\theta_K}\right). \tag{2}$$

Propagating the error from measurement of mass squared,

$$\sigma^2_{m_K} = \frac{\sigma_{m_K^2}}{4m_K^2}. \tag{3}$$

Thus this simple argument shows that $\sigma_{m_K} \sim p^2$, favoring lower momenta. However, using chambers to measure beam momentum is guaranteed to introduce a huge error, so we must rely upon measurement of all 3 ring radii: pion, kaon, and proton.

We can then rewrite Equation 1 as follows:

$$m_K^2 = m_\pi^2 + \Delta_{p\pi}\frac{\theta_\pi^2 - \theta_K^2}{\theta_\pi^2 - \theta_p^2}. \tag{4}$$

The partial derivatives become

$$\begin{aligned}
\frac{1}{2}\frac{\partial}{\partial\theta_\pi}m_K^2 &= \Delta_{p\pi}\theta_\pi \cdot \frac{\theta_K^2 - \theta_p^2}{(\theta_\pi^2 - \theta_p^2)^2} = p^2\theta_\pi \cdot \frac{\Delta_{pK}}{\Delta_{p\pi}} \\
\frac{1}{2}\frac{\partial}{\partial\theta_K}m_K^2 &= -\Delta_{p\pi}\frac{\theta_K}{\theta_\pi^2 - \theta_p^2} = p^2\theta_K \\
\frac{1}{2}\frac{\partial}{\partial\theta_p}m_K^2 &= \Delta_{p\pi}\theta_p \cdot \frac{\theta_\pi^2 - \theta_K^2}{(\theta_\pi^2 - \theta_p^2)^2} = p^2\theta_p\frac{\Delta_{K\pi}}{\Delta_{p\pi}}.
\end{aligned}$$

Hence,

$$\frac{1}{4}\sigma^2_{m_K^2} = p^4 \cdot \left[\theta_\pi^2\left(\frac{\Delta_{pK}}{\Delta_{p\pi}}\right)^2\sigma^2_{\theta_\pi} + \theta_K^2\sigma^2_{\theta_K} + \theta_p^2\left(\frac{\Delta_{K\pi}}{\Delta_{p\pi}}\right)^2\sigma^2_{\theta_p}\right] \tag{5}$$

Thus, again the error on kaon mass is proportional to momentum squared, so we are better off making the measurement at lower momentum.

To calculate the optimal fraction of pion, kaon, and proton triggers, let's make the assumption that the width of a given ring comes from dispersion of CO_2 and momentum slice of the beam. Since

$$\theta_i = \sqrt{2\left(1 - \frac{1}{n\beta_i}\right)},$$

it follows that the width of the distribution of RICH rings for particle species i is

$$w\theta_i{}^2 = \left(\frac{1}{\theta_i n^2\beta_i}\right)^2\sigma_n^2 + \left(\frac{m_i^2}{\theta_i n\beta_i^2 p^2}\right)^2(\sigma_p/p)^2. \tag{6}$$

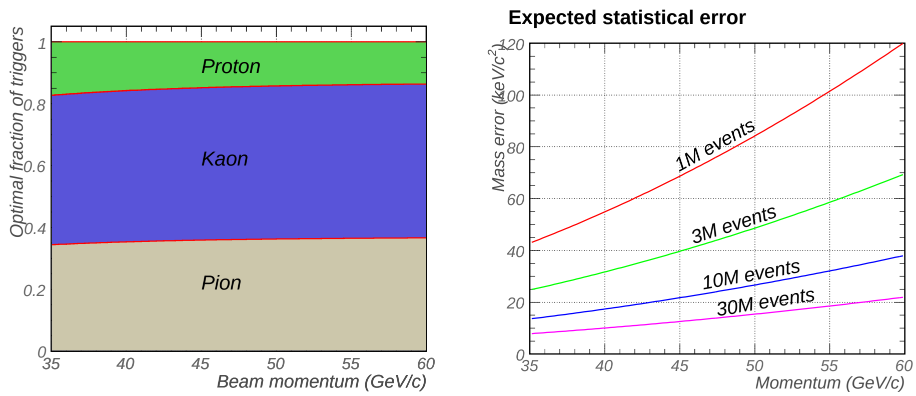

Figure 1: Optimal fraction of pion, kaon, and proton triggers (left), and expected statistical error for different total number of triggers, assuming optimal distribution of events in the sample (right).

We can assume that

$$\sigma^2_{\theta_i} = w_i^2/N_i. \tag{7}$$

It is simple to show that if $a + b + c = 1$, then the quantity

$$S = A^2/a + B^2/b + C^2/c,$$

is minimized when

$$\begin{cases} a &= A/(A+B+C) \\ b &= B/(A+B+C) \\ c &= C/(A+B+C) \end{cases} \tag{8}$$

Using Equations 5, 6, 7, and 8, we can determine the optimal fraction of pion, kaon, and proton triggers as a function of momentum. This distribution and expected statistical error are shown in Figure 1. These distributions were drawn with the following assumptions:

$$\begin{aligned} \sigma_n &= 7 \times 10^{-6} \\ \sigma_p/p &= 0.01. \end{aligned}$$

Note that with the assumed σ_p/p, the full momentum bite is $\pm 4\%$. Under these assumptions, w_{θ_i} are in pretty good agreement with observed width of ring distributions.

2 Conclusion

Based on presented derivation, 40 GeV/c seems to be the optimal momentum to make kaon mass measurement. Existing data shows

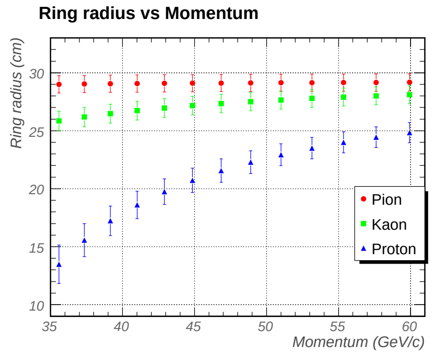

Figure 2: Dependence of pion, kaon, and proton RICH ring radii on momentum. Above ≈ 50 GeV/c, kaon and pion rings start to overlap, suggesting that it is best to do the measurement at lower momentum.

that 35 GeV/c proton rings are very wide and have few PMT hits, hence momentum close to proton threshold is likely to introduce more trouble than gain. Figure 2 shows another reason for favoring momenta below ≈ 50 GeV/c: above this momentum kaon and pion rings start to overlap.

Another reason for lower momenta is the fraction of kaons and pions in positive secondary beam. With higher momenta both of those particles are minority particles, and therefore require higher rates and potentially cause more spray in the detector.